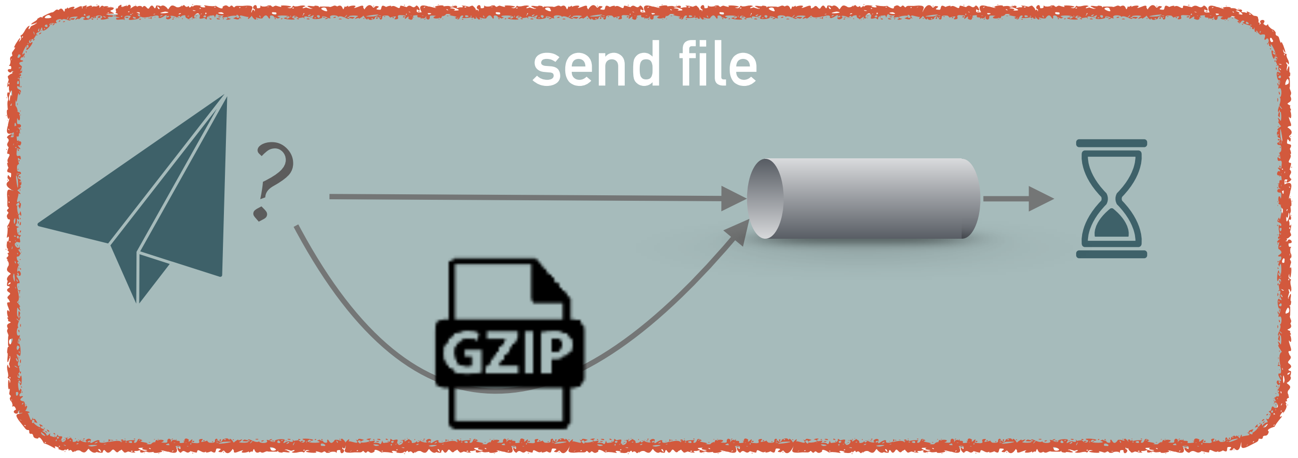
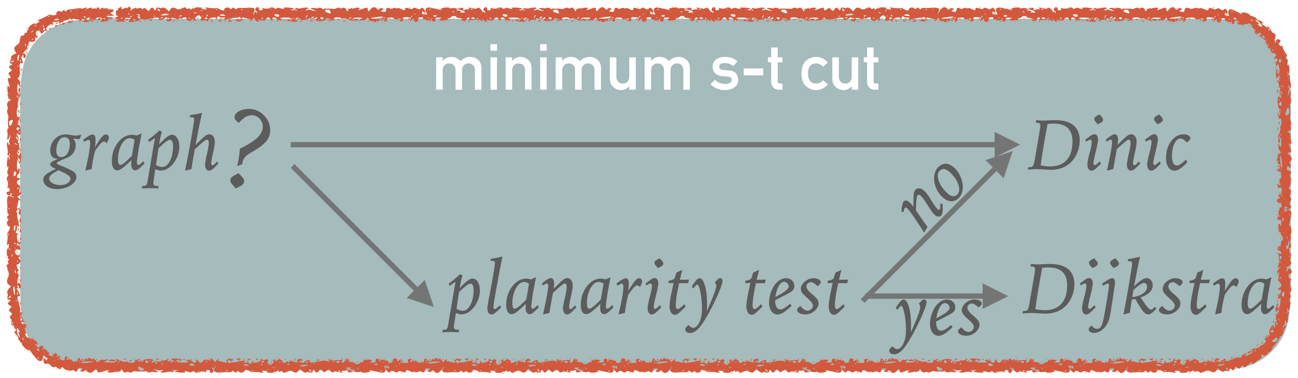
SCHEDULING WITH EXPLORABLE UNCERTAINTY

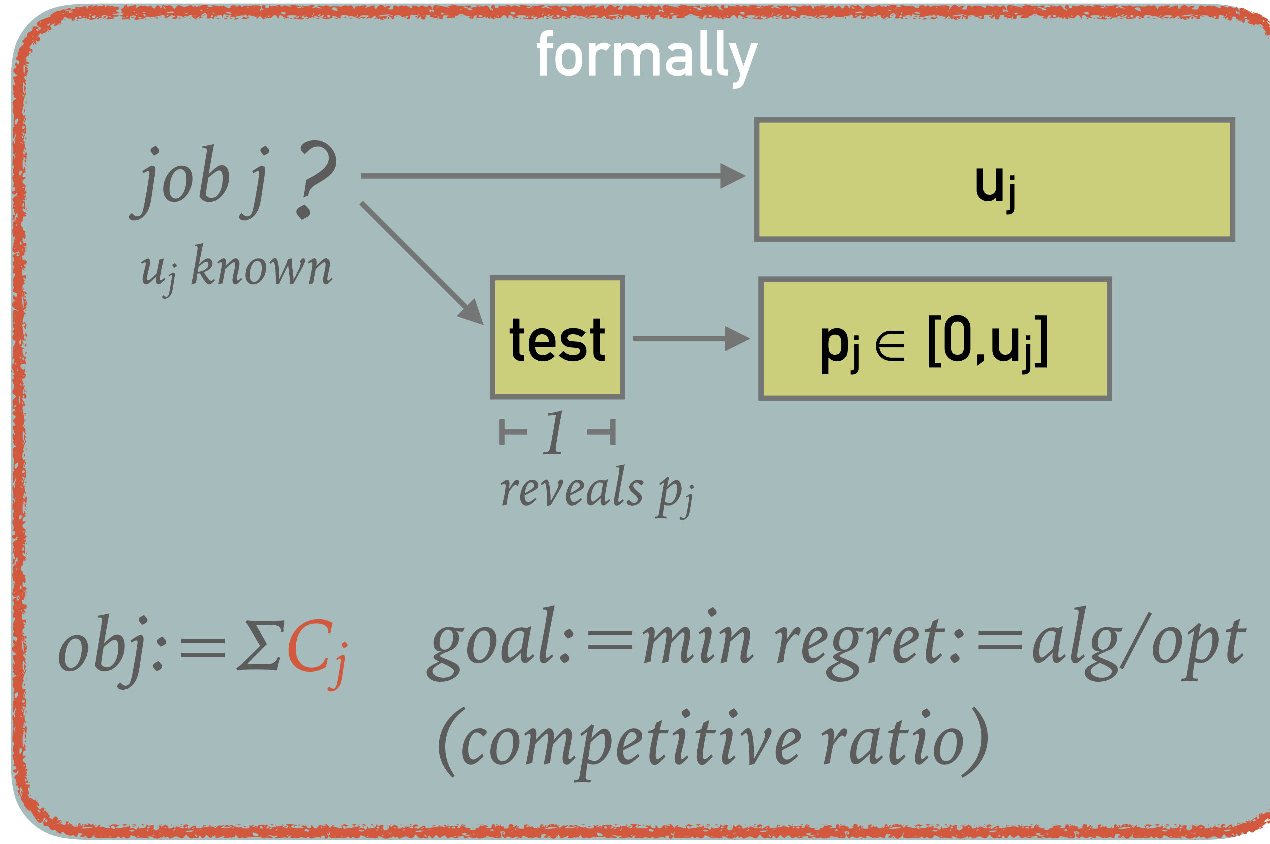
C. Dürr (Sorbonne University, CNRS)
Thomas Erlebach (Leicester)
Nicole Megow (Bremen)
Julie Meißner (Berlin)

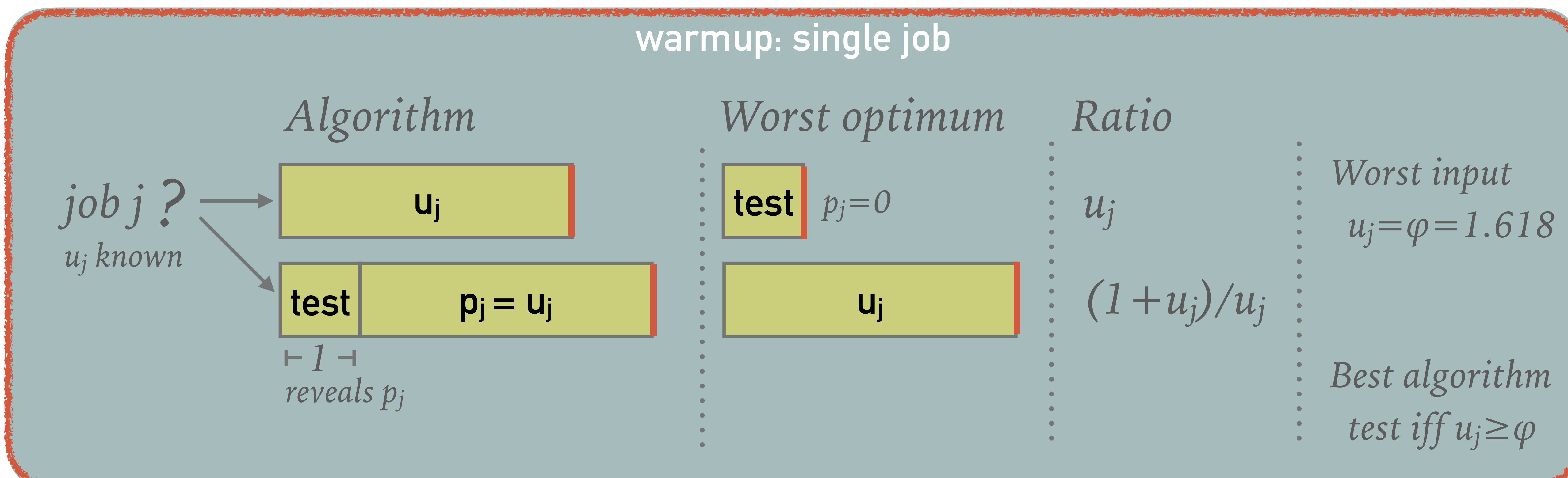
aussois scheduling workshop 2018

INTRODUCTION





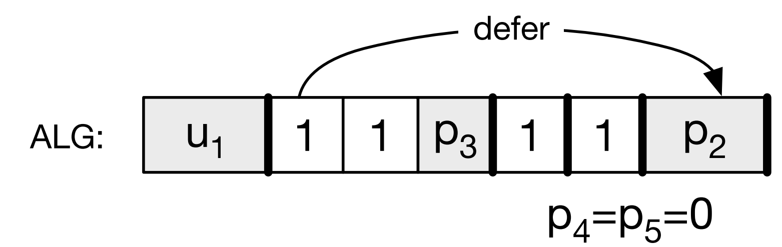




RESULTS

competitive ratio	lower bound	upper bound	algorithm
deterministic ratio	1.8546	2	THRESHOLD
randomized ratio	1.6257	1.7453 (asymptotic ratio)	RANDOM
det. ratio. when $\forall j$: $u_j=p$	1.8546	1.9338	BEAT
det. ratio. when $\forall j$: $u_j=p$, $p_j \in \{0,p\}$	1.8546	1.8668	UTE

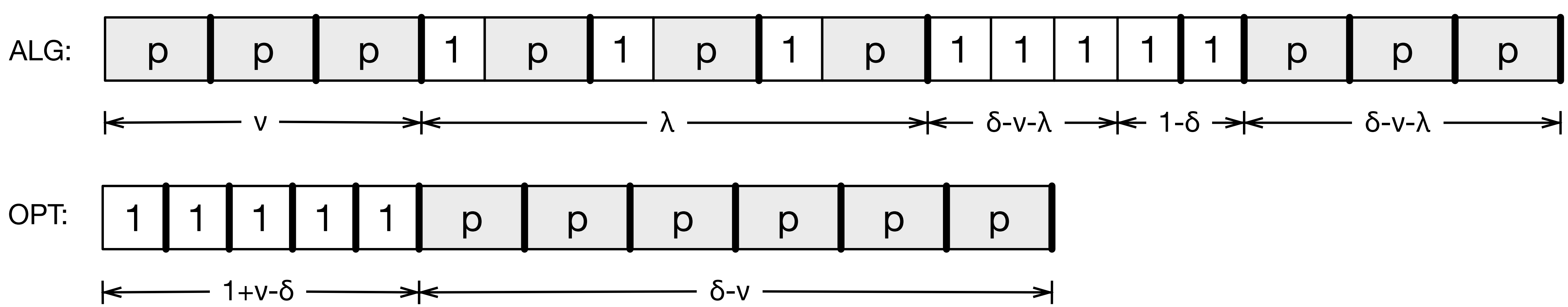
more jobs



DETERMINISTIC LOWER BOUND

competitive ratio	lower bound	upper bound	algorithm
deterministic ratio	1.8546	2	THRESHOLD
randomized ratio	1.6257	1.7453 (asymptotic ratio)	RANDOM
det. ratio. when $\forall j$: $u_j=p$	1.8546	1.9338	BEAT
det. ratio. when $\forall j$: $u_j=p$, $p_j \in \{0,p\}$	1.8546	1.8668	UTE

- n uniform jobs with upper limit p
- Index jobs in order they are touched by algorithm (tested or executed untested)
- $p_j=0$ if $j \geq \delta n$ or job j is executed untested by algo. $p_j=p$ otherwise
- Algorithm gets even to know δ
- Any decent algorithm produces a schedule with above structure for parameters v, λ with $v+\lambda \leq \delta$
- The competitive ratio is ALG(δ,v, λ,n) / OPT(δ,v,n)
- Algorithm (minimizer) chooses v,λ
- Adversary (maximizer) chooses n,δ
- Analyzing local optima yields ratio 1.854628

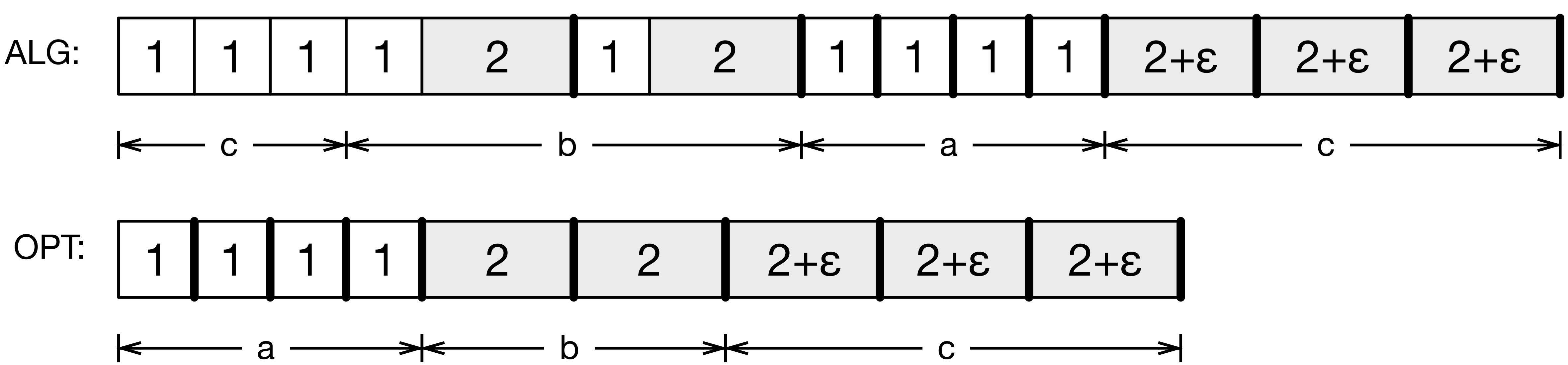


ALGORITHM THRESHOLD

competitive ratio	lower bound	upper bound	algorithm
deterministic ratio	1.8546	2	THRESHOLD
randomized ratio	1.6257	1.7453 (asymptotic ratio)	RANDOM
det. ratio. when $\forall j: u_j = p$	1.8546	1.9338	BEAT
det. ratio. when $\forall j: u_j = p, p_j \in \{0,p\}$	1.8546	1.8668	UTE

- Execute untested all jobs j with $u_j \leq 2$ in order…
- Test all other jobs in arbitrary order. If $p_j \leq 2$, execute, otherwise defer.
- Execute all deferred jobs in order…

- Worst case instance:
  a jobs $u_j=2, p_j=0$
  b jobs $u_j=p_j=2$
  c jobs $u_j=p_j=2+\varepsilon$
- Simple arithmetics: $ALG(a,b,c) \leq 2 \cdot OPT(a,b,c)$

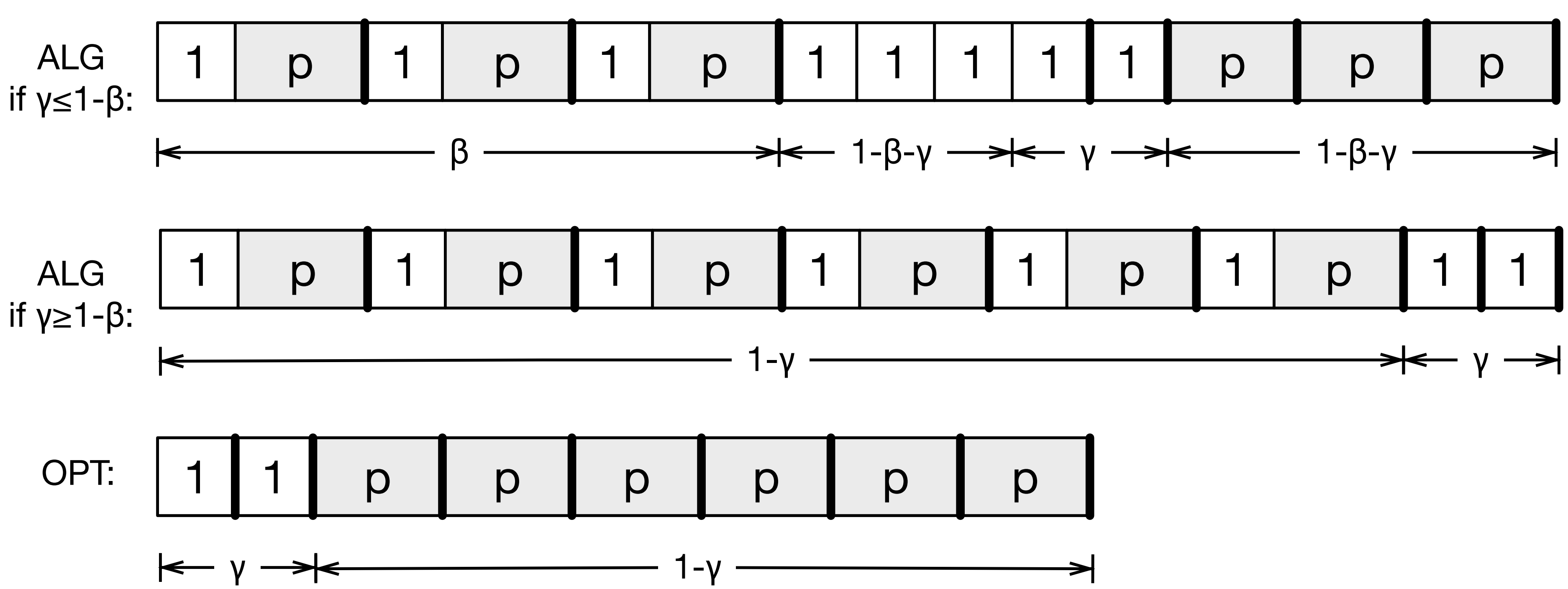


ALGORITHM UTE

competitive ratio	lower bound	upper bound	algorithm
deterministic ratio	1.8546	2	THRESHOLD
randomized ratio	1.6257	1.7453 (asymptotic ratio)	RANDOM
det. ratio. when $\forall j$: $u_j=p$	1.8546	1.9338	BEAT
det. ratio. when $\forall j$: $u_j=p$, $p_j \in \{0,p\}$	1.8546	1.8668	UTE

- has ratio $\rho = \frac{1+\sqrt{3+2\sqrt{5}}}{2} \approx 1.8668.$
- Parameter $\beta = \dfrac{1-\bar{p}+\bar{p}^2-\rho+2\bar{p}\rho-\bar{p}^2\rho}{1-\bar{p}+\bar{p}^2-\rho+\bar{p}\rho}$
- Execute all jobs untested if $p \leq \rho$
- Otherwise test all jobs. Execute right after their test the first max{0,β} fraction of jobs. Then only if $p_j=0$. Finally execute deferred jobs.

- Worst case instance defined by length p fraction γ: the first γn tested jobs have $p_j=p$ and the remaining $p_j=0$
- Second order analysis to optimize p,γ and β



ALGORITHM RANDOM

competitive ratio	lower bound	upper bound	algorithm
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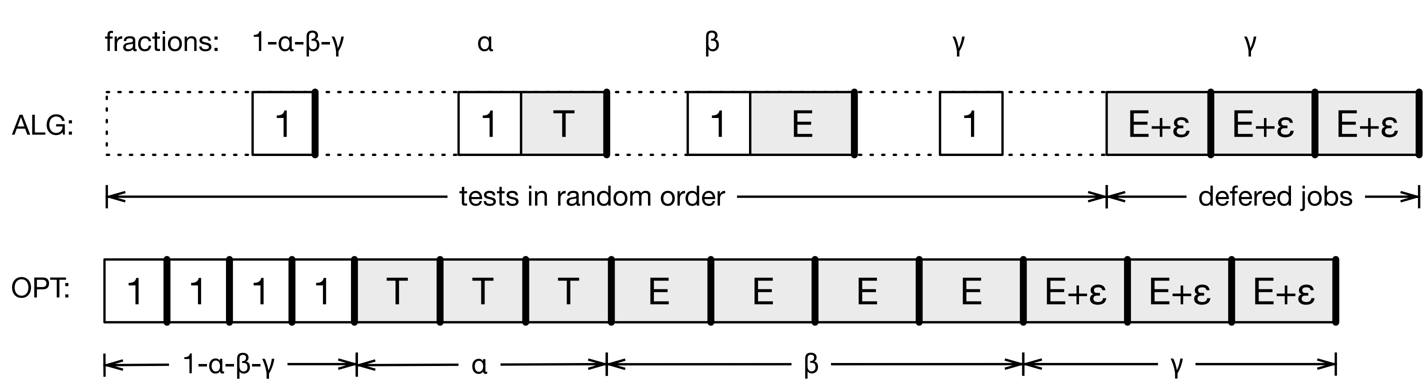

- has parameters $T \geq E$

- Schedule untested all jobs with upper limit < T in increasing upper limit order
  Test in random order all larger jobs j,
  if $p_j \leq E$ execute immediately, else defer their execution
  Finally schedule deferred jobs
  in increasing processing time order

- Worst case instances:
  (1-α-β-γ) fraction of jobs : $u_j=T$, $p_j=0$
  αn jobs have $u_j=T$, $p_j=T$
  βn jobs have $u_j=E$, $p_j=E$
  γn jobs have $u_j=E+\varepsilon$, $p_j=E+\varepsilon$

- Ratio $\leq$ T iff
  $G := OPT \cdot T - ALG \geq 0$

- Algorithm chooses T, E to max. G
  Adversary chooses α,β,γ to min. G

« Every decision we make,
is the wrong one.

-Murphy