ONLINE MAXIMUM MATCHING WITH RECOURSE

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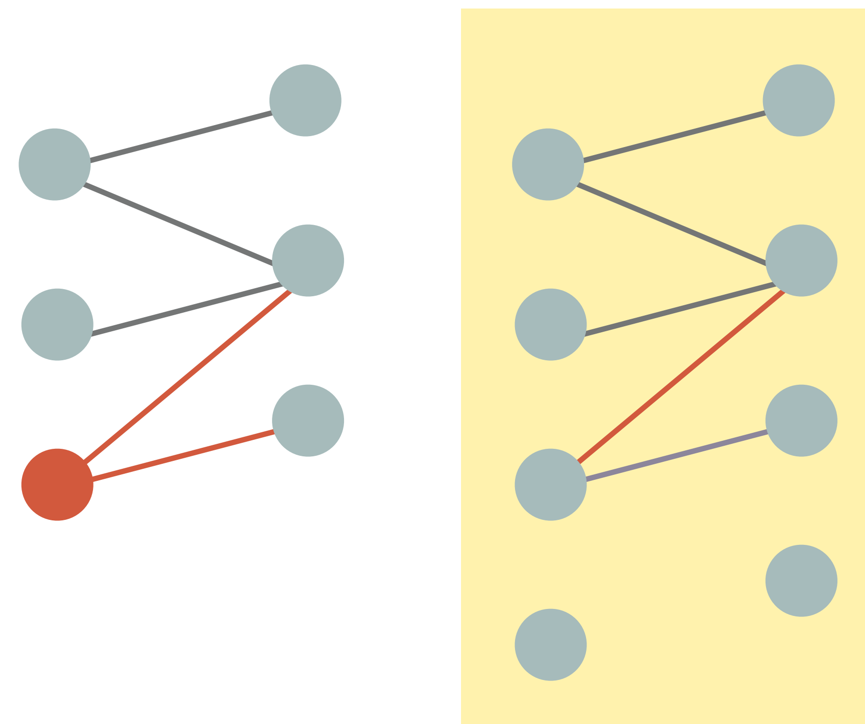
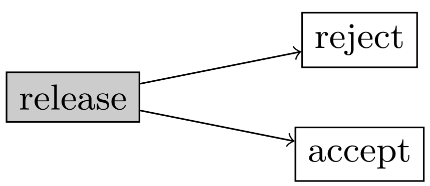
ONLINE MATCHING MODELS

maximum cardinality or maximum weight

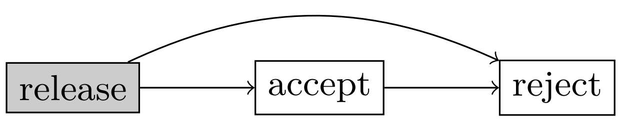
either vertex- or edge-arrival



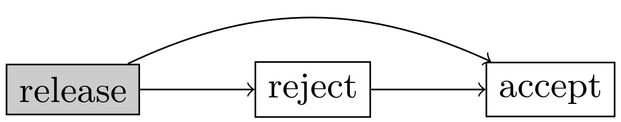
recourse models restricting edges



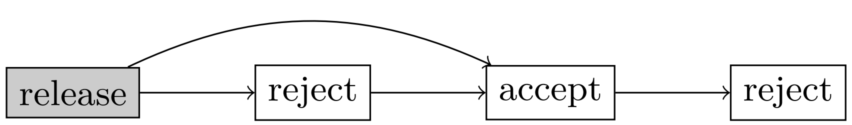
Standard model



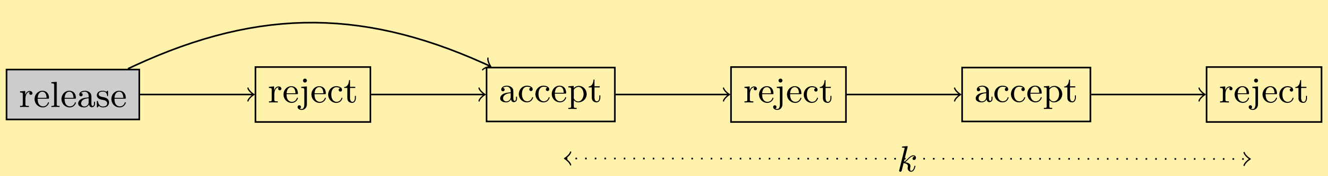
Late reject model, also called free disposal or preemptive model



Late accept model, also called edge 1-bounded recourse model



Late accept/reject model, also called edge 2-bounded recourse model



What is the competitive ratio in function of k ?

PREVIOUS RESULTS

maximum cardinality, edge-arrival

Different recourse models	determinstic	randomized
release → reject; release → accept Standard model	=2	≥1.690 ≤ 1.8 (for trees)
release → accept → reject Late reject model, also called free disposal or preemptive model	=2	≥1.693 =1.333 (on paths)
release → reject → accept Late accept model, also called edge 1-bounded recourse model	=2	
release → reject → accept → reject Late accept/reject model, also called edge 2-bounded recourse model	=1.5	
release → reject → accept → reject → accept → reject k	≥1+1/k ≤1+O(log k/k)	≥1+1/(9k-1)

PREVIOUS RESULTS

maximum cardinality, edge-arrival

Different recourse models	determinstic	randomized
release → reject release → accept Standard model	Greedy	[Buchbinder,Segev, Tkach'2017]
release → accept → reject Late reject model, also called free disposal or preemptive model		[Epstein,Levin,Segev, Weimann'2013] [Chiplunkar,Tirodkar, Vishwanathan'2015]
release → reject → accept Late accept model, also called edge 1-bounded recourse model		
release → reject → accept → reject Late accept/reject model, also called edge 2-bounded recourse model	[Boyar, Favrholdt, Kotrbčík, Larsen, 2017]	
release → reject → accept → reject → accept → reject k	[Avitabile, Mathieu, Parkinson, 2013]	

OUR MODEL

- Fixed edge recourse budget k
- Every edge has type 0 at arrival
- Whenever edge enters or leaves the matching, its type is increased
- Edge is blocked when its type is k
- Appl ng an augmenting path

Quite bad when k is odd

hence we focus mostly on even k

SIMPLE LOWER BOUND 1+1/K

- Example: k=2

- Algorithm has to augment.

- Adversary extends the path.

- Algorithm has to augment. Adversary extends the path. Algorithm cannot augment. Ratio is (k+1)/k.

blocked edge

IMPROVED LOWER BOUND 1+1/(K−1) FOR ALL K≥3

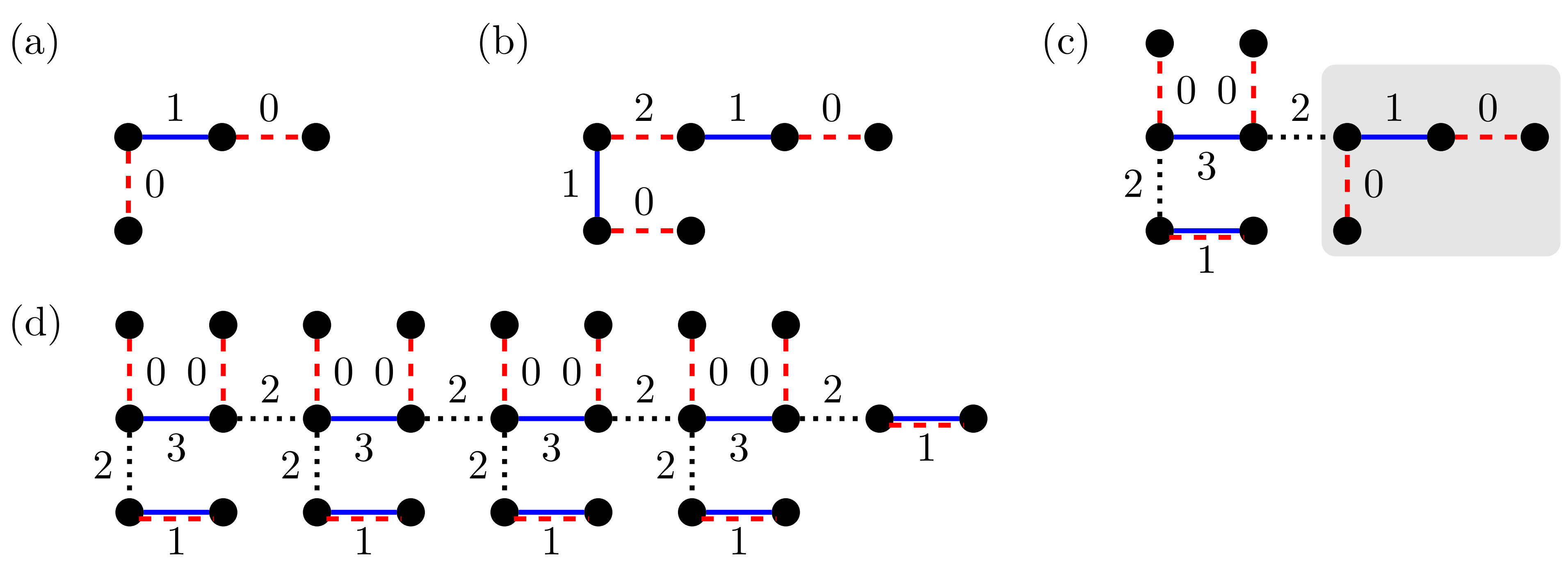
- For example k=3. Consider algorithm claiming ratio 3/2-ε. Initially release 1 then 2 edges.

(a) Ratio=2, algorithm needs to augment.

(b) Ratio=3/2, algorithm needs to augment.

(c) Construction is repeated,

(d) until ratio (3n+1)/(2n+1) exceeds 3/2-ε.



TWO ALGORITHMIC IDEAS

Apply augmenting paths as late as possible [AMP'2013]

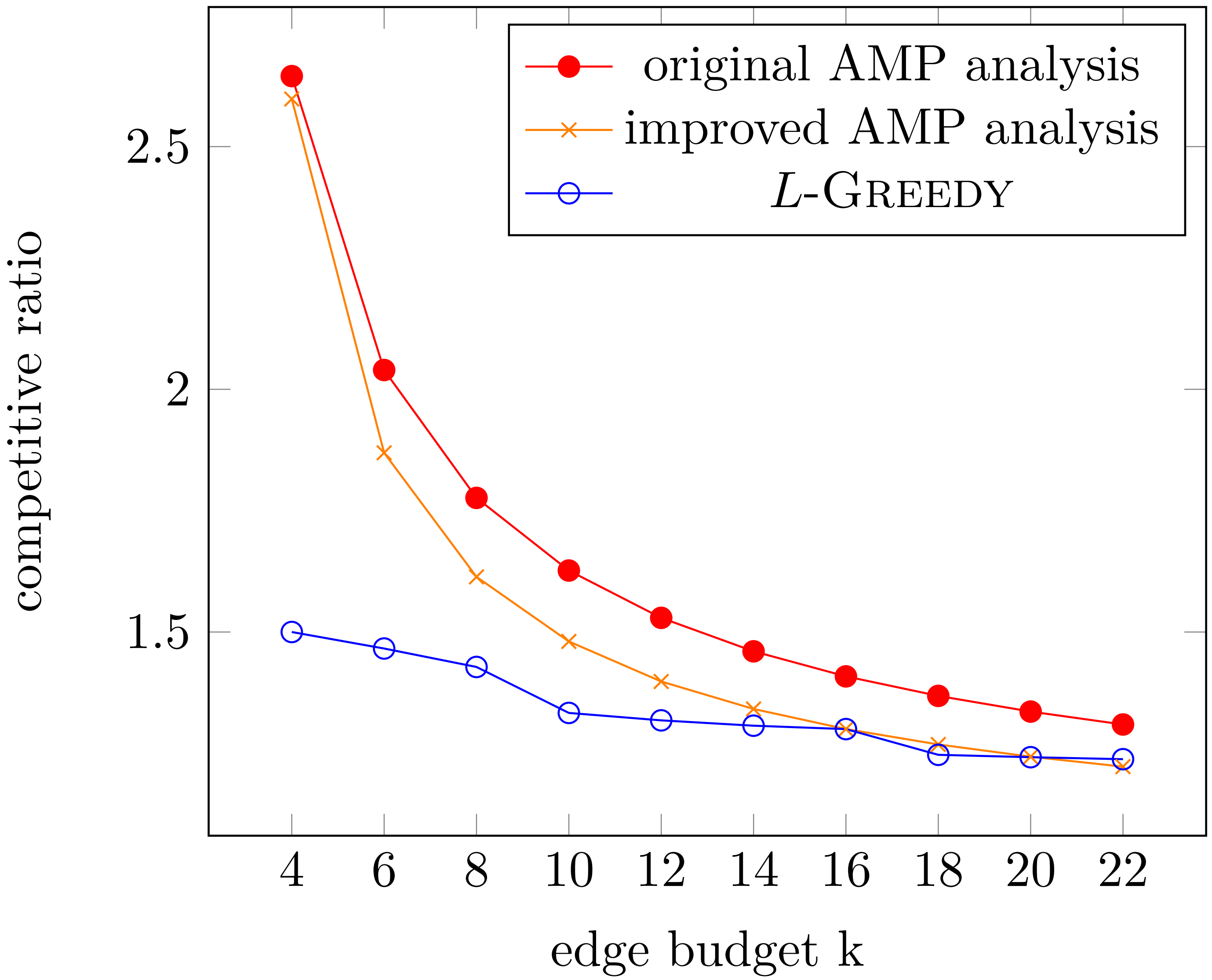
- Generic Lazy algorithm: claims ratio R, is idle whenever ratio is ≤ R
- Hard to analyze, so doubling algorithm **AMP** is studied instead: Whenever OPT exceeds next integer power of some ρ, apply all possible augmenting paths.

Apply only short augmenting paths [Our paper]

- Intuition: preserve edge budgets by minimizing global recourse
- **L-Greedy**: restrict to length 2L+1 augmenting paths, L is optimized at ⌊√(k-1)⌋

To do:
Analyze the combination of both techniques

COMPARING ALGORITHMS



- AMP has ratio 1+O(log k/k)
- L-Greedy has ratio 1+O(1/√k) proof by charging scheme
- Cutting point at k=20

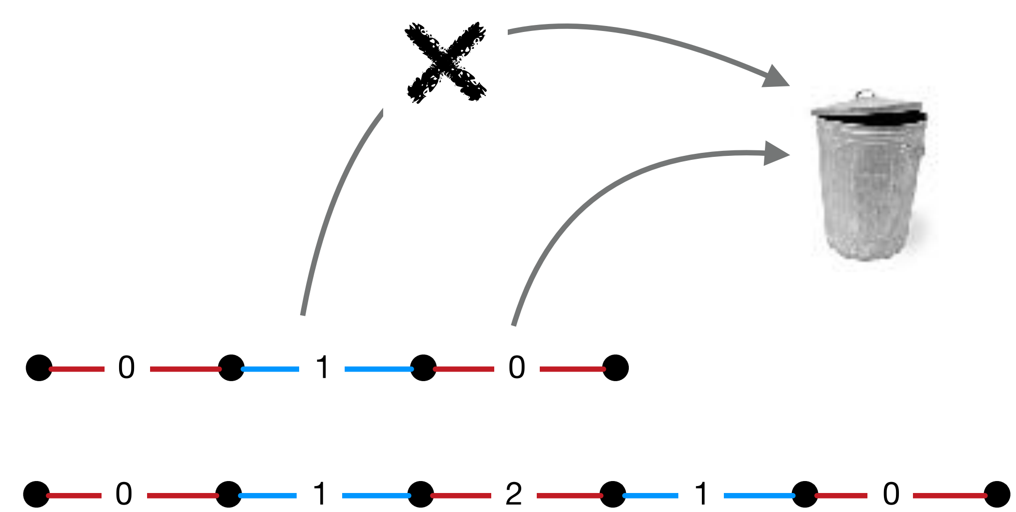
ATTEMPTS TO CLOSE THE GAP FOR K=4

- We studied a stronger adversarial model. Edges can arrive, but also depart (if not in the current matching). Adversary maintains a collection of paths. Edge types along paths form strings on {0,1,…,k} of alternating parity. Now we have a **game**: Algorithm can augment strings (increment edge types), Adversary can merge or split strings.

1.5 *upper bound, L-Greedy*

1.4285 *lower bound, arrival/departure model*

1.3333 *lower bound, arrival model*



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- thank you